Nassau County Interscholastic Mathematics League

Contest #1 Answers must be integers from 0 to 999, inclusive. 2021 – 2022

No calculators are allowed.

Time: 10 minutes **Name: _____**

1. Compute 10% of 20% of 10,000.
2. Let S be the sum of all of the 4-digit positive odd integers that use each of the digits 2, 4, 5, and 6 exactly once. When S is divided by 1000, compute the remainder.

1. ☐ 2. ☐

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3. My favorite pet store scheduled 20 dogs and 15 cats for a bath on a particular day. These pets either love or hate taking a bath. Of these animals, 14 dogs and 11 cats hate taking a bath. Given that an animal loved the bath at the store that day, compute the product of 100 and the probability that the bath lover was a dog.

4. If $\sin x + \cos x = \frac{5}{4}$ and $\sin(2x)$ in simplest terms is $\frac{p}{q}$, compute pq.
   Note: $\sin(2x) = 2\sin x \cos x$.

3. ☐ 4. ☐

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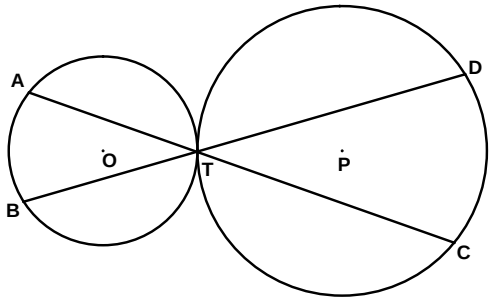
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5. The positive solution to $9^x = 81 \cdot \sqrt[x]{3}$ is written in $p + q\sqrt{r}$ form, where p, q, and r are in simplest rational form, and $\sqrt{r}$ is in simplest radical form. Compute pqr.

6. Circles O and P are externally tangent at point T. Chords $\overline{AT}$ and $\overline{BT}$ are extended through point T to points C and D, respectively. If $TA = 2,\ TB = 3,\ TC = 5$, and in simplest form, TD can be expressed as $\frac{p}{q}$, compute $p + q$.



5.

6.

Solutions for Contest #1

1. 20% of 10000 is 2000. 10% of 2000 is **200.**

2. The sum of the six integers is $2465+2645+4265+4625+6245+6425=26670$. The required remainder is **670**.
   Alternatively, The digit 5 is always in the ones column and each of the other digits appear twice in the six possible arrangements. Therefore, the sum of the numbers in the ones column is $6(5)=30$ and the sum in each of the other three columns will be $2(2+4+6)=24$. The total is $24(1000)+24(100)+24(10)+30=26670$. Divide this by 1000 to get a remainder of **670**.

3. 6 dogs and 4 cats love taking a bath. The required product is $100 \cdot 0.6 = \mathbf{60}$.

4. $\sin x + \cos x = \frac{5}{4} \rightarrow (\sin x)^2 + (\cos x)^2 + 2\sin x \cos x = \frac{25}{16}$.
   Since $(\sin x)^2 + (\cos x)^2 = 1$, $2\sin x \cos x = \frac{9}{16} \rightarrow \sin(2x) = \frac{9}{16}$.
   The required product is **144**.

5. $9^x = 81 \cdot \sqrt[x]{3} \rightarrow 3^{2x} = 3^4 \cdot 3^{\frac{1}{x}} \rightarrow 3^{2x} = 3^{4+\frac{1}{x}} \rightarrow 2x = 4 + \frac{1}{x} \rightarrow 2x^2 - 4x - 1 = 0 \rightarrow x = \frac{4 \pm \sqrt{24}}{4} = 1 \pm \frac{1}{2}\sqrt{6} \rightarrow p = 1,\ q = \frac{1}{2},$ and $r = 6$. The required product is **3**.

6. Draw $\overleftrightarrow{XTY}$, the common internal tangent. Also draw $\overline{AB}$ and $\overline{CD}$. Since $\measuredangle ATX \cong \measuredangle YTC$, $m\measuredangle ATX = \frac{1}{2}m\widehat{AT}$, and $m\measuredangle CTY = \frac{1}{2}m\widehat{CT}, m\widehat{AT} = m\widehat{CT}$. Therefore, $\measuredangle B \cong \measuredangle D$. Since $\measuredangle ATB \cong \measuredangle CTD, \Delta BTA \sim \Delta DTC \rightarrow \frac{AT}{BT} = \frac{CT}{DT} \rightarrow \frac{2}{3} = \frac{5}{DT} \rightarrow DT = \frac{15}{2}$ and the required sum is **17**.

